Int. J. Heat Mass Transfer, Vol. 23, pp. 73–87

0017-9310/80/0101-0073 $02.00/0

HEAT-TRANSFER CORRELATIONS FOR NATURAL CONVECTION BOILING

K. STEPHAN and M. ABDELSALAM
Universität Stuttgart, Stuttgart, Germany

(*Received* 14 *December* 1978)

Abstract—To-date there exists no comprehensive theory allowing the prediction of heat-transfer coefficients in natural convection boiling, in spite of the many efforts made in this field. In order to establish correlations with wide application, the methods of regression analysis were applied to the nearly 5000 existing experimental data points for natural convection boiling heat transfer. As demonstrated by the analysis, these data can best be represented by subdividing the substances into four groups (water, hydrocarbons, cryogenic fluids and refrigerants) and employing a different set of dimensionless numbers for each group of substances, because certain dimensionless numbers important for one group of substances are unimportant to another. One equation valid for all substances could be built up, but its accuracy would be less than that obtained for the individual correlations without adding undesirable complexity.

NOMENCLATURE

a, thermal diffusivity [m^2/s];
A, heater surface [m^2];
b, $[2\sigma/g(\rho'-\rho'')]^{1/2}$ Laplace constant;
c_p, specific heat capacity at constant pressure in [kJ/(kg · K)];
d, equilibrium break-off-diameter $d = 0.146\beta b$ [m];
f, bubble frequency [1/s];
g, acceleration of gravity [m/s^2];
p, pressure [bar];
p_c, critical pressure [bar];
$\dot{q}$, heat flux density [W/m^2];
r, enthalpy of evaporation [kJ/kg];
R_p, mean roughness according to DIN (Deut. Ind. Norm) 4762;
T, thermodynamic temperature [K];
T_w, wall temperature [K];
T_s, saturation temperature [K];
ΔT, $T_w - T_s$, difference between wall and saturation temperature [K].

Greek symbols

α, $\dot{q}/\Delta T$, heat-transfer coefficient [W/(K · m^2)];
β, contact angle [deg];
λ, heat conductivity [W/(K · m)];
ν, kinematic viscosity [m^2/s];
ρ, mass density [kg/m^3];
σ, surface tension [N/m].

Subscripts and superscripts

$'$, saturated liquid;
$''$, saturated vapour;
c, cover, or heater surface if surface is unprotected;
s, solid material behind cover, at saturation temperature.

1. INTRODUCTION

HEAT transfer in boiling has been investigated intensively for many years and many phenomena have been explained. However, at present it is still difficult or even impossible to predict heat-transfer coefficients with satisfactory accuracy. Many of the existing results on simple phenomena are inconsistent with each other and should be critically reviewed. One of the tasks still to be completed is the review in a comprehensive and critical way of the existing data on heat transfer in natural convection boiling and the correlation of these data by equations. The present paper is directed towards this object.

When attempting a general correlation of many experimental data, various procedures are conceivable: one can, for example, determine what correlation among the many known from the literature represents best the entity of all existing data and then if necessary improve this correlation, or, one can build up a model, derive the general form of an equation from it and then adapt the constants and exponents in this equation to the experimental data. However, taking into account the present knowledge on heat transfer in natural convection boiling, both of these procedures are likely to prove unsatisfactory. All of the existing equations are based on models which prove nonsuitable for some substances, apparently none take into account all of the processes important to boiling heat transfer, and present knowledge is not sufficient for the building up of a valid general model. Consequently, it does not seem appropriate to start from a given model, but correlate instead the existing experimental data by means of more mathematical methods.

2. CORRELATIONS BY MEANS OF THE REGRESSION ANALYSIS

In order to arrive at equations for heat transfer in natural convection boiling it is reasonable to start from the fact that a certain number of physical properties and variables characterize the heat-transfer process. Such properties and variables are,

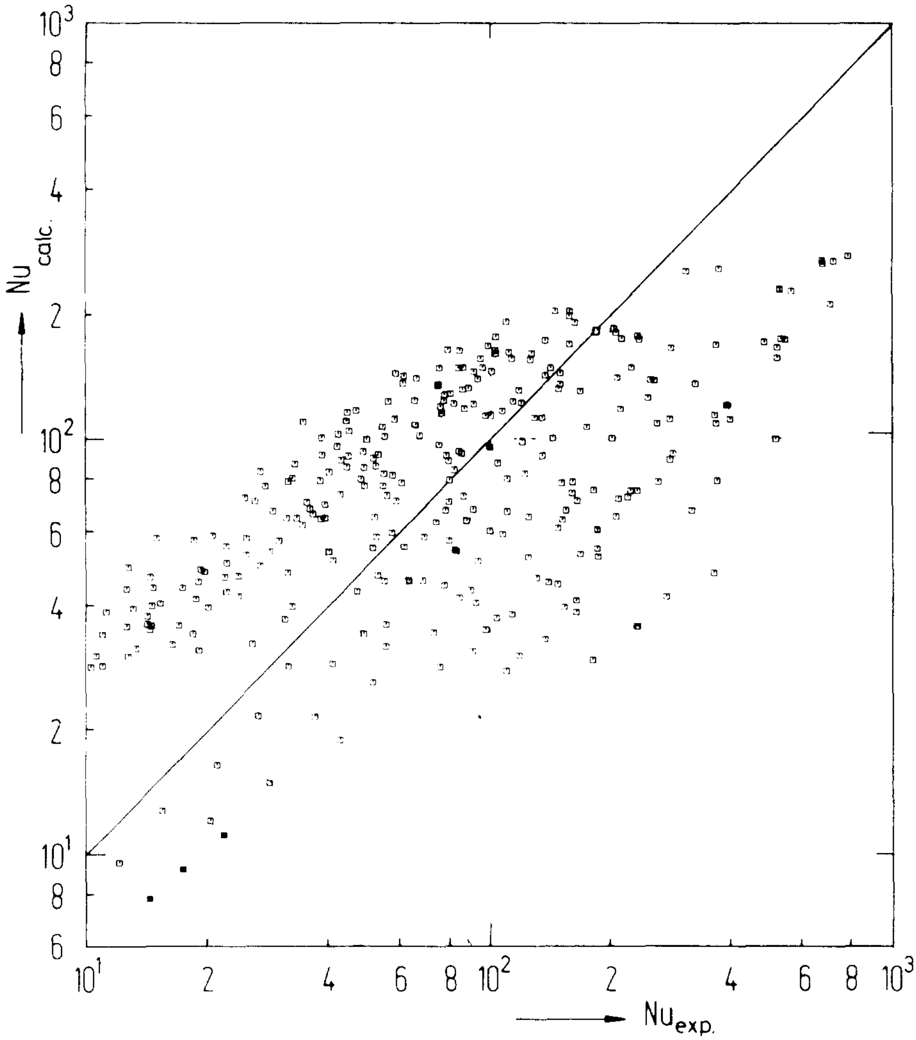

FIG. 1. Nusselt number for water after first step of regression analysis.

e.g. [1], the variables $\dot{q}$, $T_w - T_s$, f, g, R_p, d, T_s, the fluid physical properties λ', ρ' c'_p, ρ'', r, η, σ the thermal properties of the heater ρ_s, c_{ps}, λ_s and also those of a cover material ρ_c, c_{pc}, λ_c, that protects the heater surface. These properties may be combined in the usual way to yield a set of dimensionless numbers. A possible set is [1]:

$$X_1 = (\dot{q}d)/(\lambda' T_s);\ X_2 = (a'^2\rho')/(\sigma d);$$
$$X_3 = (c'_p T_s d^2)/a'^2;\ X_4 = (rd^2)/a'^2;\ X_5 = \rho''/\rho';$$
$$X_6 = v'/a';\ X_7 = a'^2/(d^3 g);\ X_8 = R_p/d;$$
$$X_9 = (\rho c_p \lambda)_c/(\rho' c'_p \lambda');$$
$$X_{10} = (\rho c_p \lambda)_s/(\rho' c'_p \lambda');\ X_{11} = a_c/a';\ X_{12} = a_s/a'$$
$$\text{and } Y = \dot{q}d/[(T_w - T_s)\lambda'].$$

The Nusselt number $Y = (\alpha d)/\lambda' = Nu$ and the dimensionless numbers X_i depend on each other

$$Y = f(X_1, X_2, \ldots). \qquad (1)$$

For the following considerations, this set of dimensionless numbers need not necessarily be complete. It is required only that the physical properties essential for heat transfer in natural convection boiling be included in the above dimensionless numbers, an assumption that seems to be fulfilled since the list of thermal properties certainly contains all those properties that have proved to be relevant for heat transfer in natural convection boiling.

A very powerful tool for finding a correlation between the Nusselt number and the values X_i is given by regression analysis, which proved to be very useful in statistical economics [2, 3]. Recently Wagner [4] applied this method to obtain a vapour pressure equation from experimental data. The regression analysis represents a method for deriving a correlation between an independent and several dependent variables. It is based on two assumptions:

(i) A sufficiently large number of experimental data describing the influence of the essential variables over a wide range must be available. The quality of the correlation depends decisively on the number and accuracy of the experimental data.

(ii) A general form of equation (1) must be known including all essential variables.

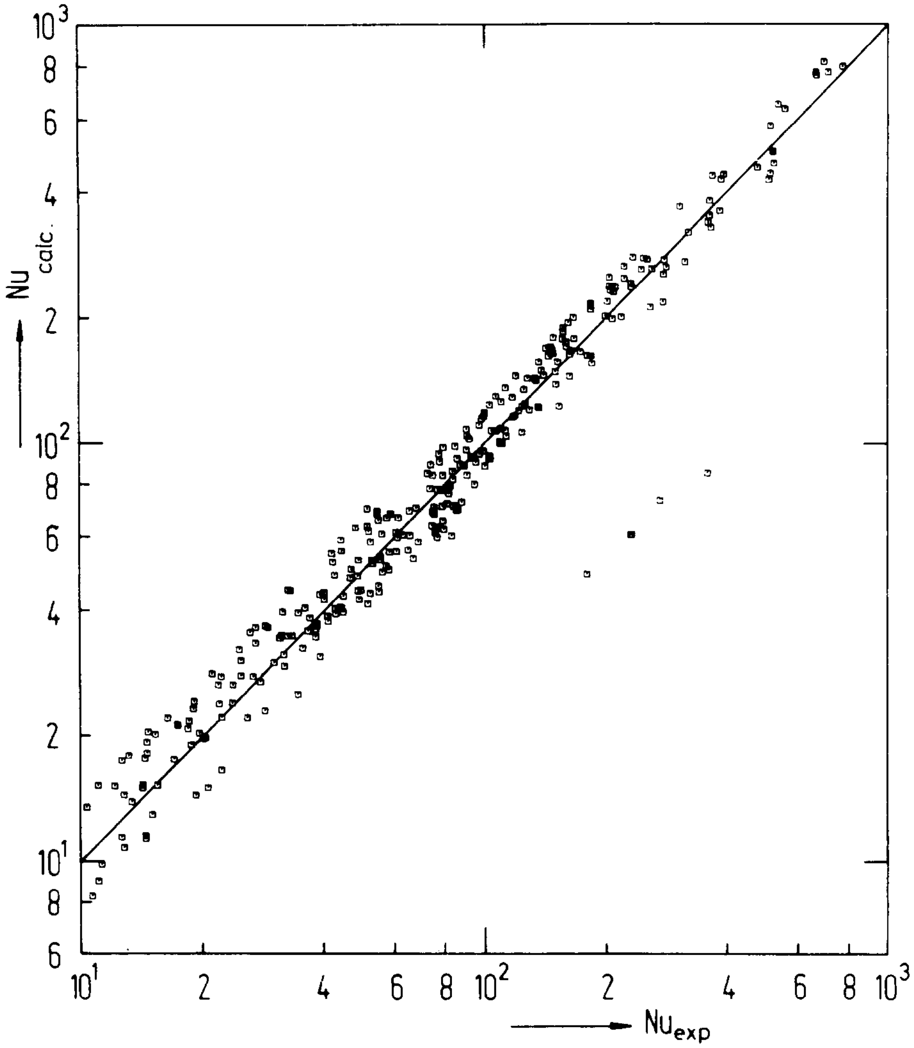

FIG. 2. Nusselt number for water after second step of regression analysis.

In fact there exists a great number of experimental data on heat transfer in natural convection boiling for many substances, especially for substances which are often used in technical applications such as water, hydrocarbons, cryogenic fluids and refrigerants. A possible form of equation (1) is a power law which has proved to be very useful in many heat-transfer problems. However, such a form has the disadvantage that the pressure dependency of heat-transfer coefficients then is mainly represented by a power of $X_5 = \rho''/\rho'$ which is not adequate over a wide pressure range. The pressure dependency can be much better described, as confirmed by the following results, by introducing an additional term $X_{13} = (\rho' - \rho'')/\rho'$ in the power law. We therefore use the following form

$$Y = e^{\beta_0} X_1^{\beta_1} X_2^{\beta_2} \dots X_{13}^{\beta_{13}}. \qquad (2)$$

Y can be whatever one defines it to be.

The regression analysis does not aim at estimating all exponents β_i. This could be done by a mere adjustment to the experiments. The regression analysis rather allows to select those values X_i, which exert the most significant influence on the dependent variable Y. This selection may be achieved in different steps according to the following scheme:

In a first step for each of the independent variables an equation of the form

$$\hat{Y} = e^{\mu_0} X_1^{\mu_1} \qquad (3)$$

is assumed, where X_1 now stands for each of the 13 variables. For each of them, the exponents μ_0 and μ_1 are evaluated according to the method of least squares. Of all the different equations employed in the analysis the one which contains the most essential dimensionless number X_1^* is the one which yields the smallest square error sum

$$Q_1 = \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2. \qquad (4)$$

In a second step for each of the remaining 12 independent variables an equation of the form

$$\hat{Y} = e^{\mu_0} X_1^{*\mu_1} X_2^{\mu_2} \qquad (5)$$

is introduced, where X_1^* is the most essential variable from the first step and X_2 stands for the other

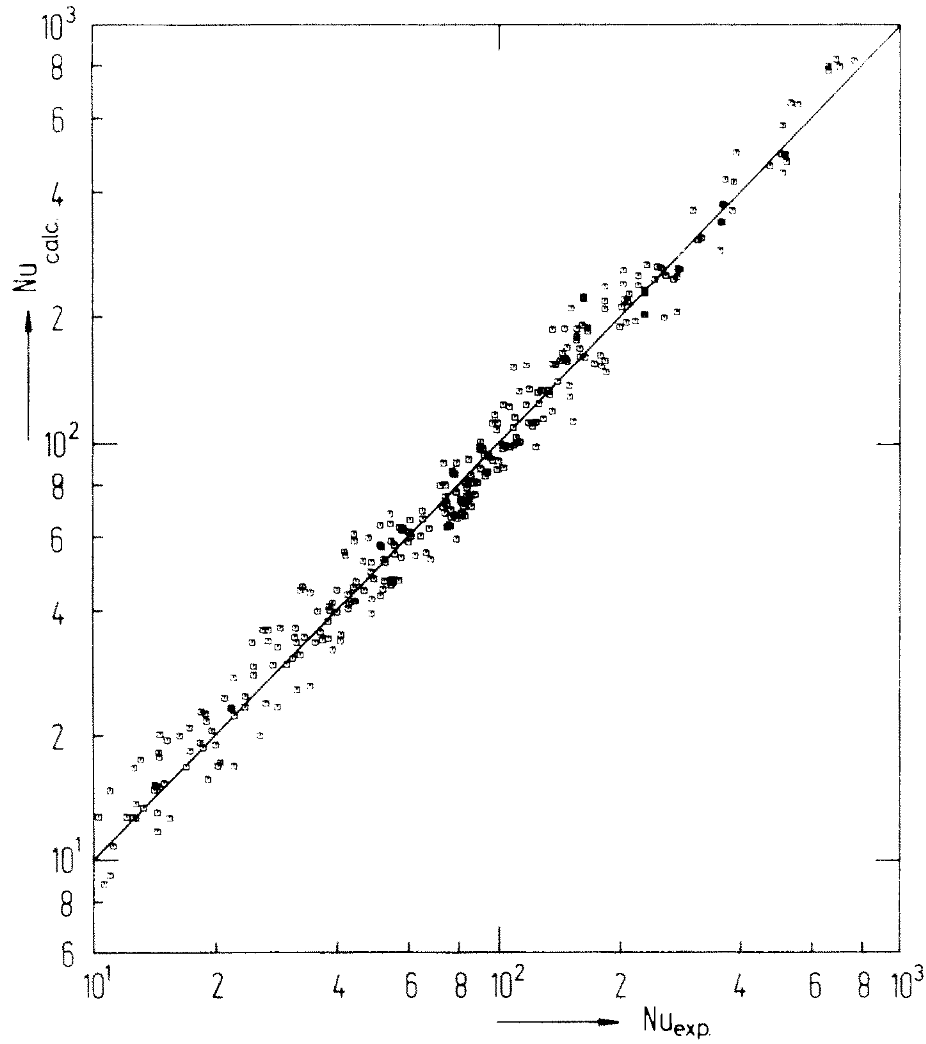

FIG. 3. Nusselt number for water after third step of regression analysis.

remaining variables. For each of these remaining variables the square error sum Q_2 is calculated and yields the next essential variable X_2^*. The procedure is continued until the experimental accuracy is well represented by the power law.

Some specific procedures, however, must be observed in the course of the analysis. From the second step on, the significance of the individual terms and also that of the actual equation must be tested again. One has to determine whether the independent variables in the actual equation, except the variable from the last step, may be replaced by one of the variables not yet included in the actual equation. Thereby the actual equation and the order of the independent variables may be changed.

As an example, Figs. 1–4 present the results for boiling of water in natural convection. The deviation between the Nusselt number Nu_{exp} calculated from experimental data and the Nusselt number Nu_{calc} from the regression analysis is considerable, if the Nusselt number is assumed to depend only on the most essential independent variable $X_1^* = (\dot{q}d)/(\lambda' T_s)$, Fig. 1. The average deviation is about 74.06%. It is reduced to 13.96%, if one includes a second variable $X_2^* = a'^2/(rd^2)$ in the analysis, Fig. 2. A further reduction of the average error to 12.2% is obtained, Fig. 3, when adding as a third variable $X_3^* = (c'_p T_s d^2)/a'^2$ and finally, Fig. 4, the average error decreases to 11.3% by taking up as a fourth variable $X_4^* = (\rho' - \rho'')/\rho'$. Introduction of further dimensionless numbers does not improve the result. In these calculations the methods of linear and those of nonlinear regression analysis [5] were applied and it turned out that the results from the linear analysis represented experimental data better than those from nonlinear analysis.

3. SELECTION OF SUBSTANCES AND DATA

In order to apply the method of regression analysis to experimental data in natural convection boiling, the existing data had to be collected and critically reviewed. This was done under the following criteria:

Only data concerning pool boiling on horizontal surfaces in the range of fully established nucleate boiling under the influence of the gravity field were

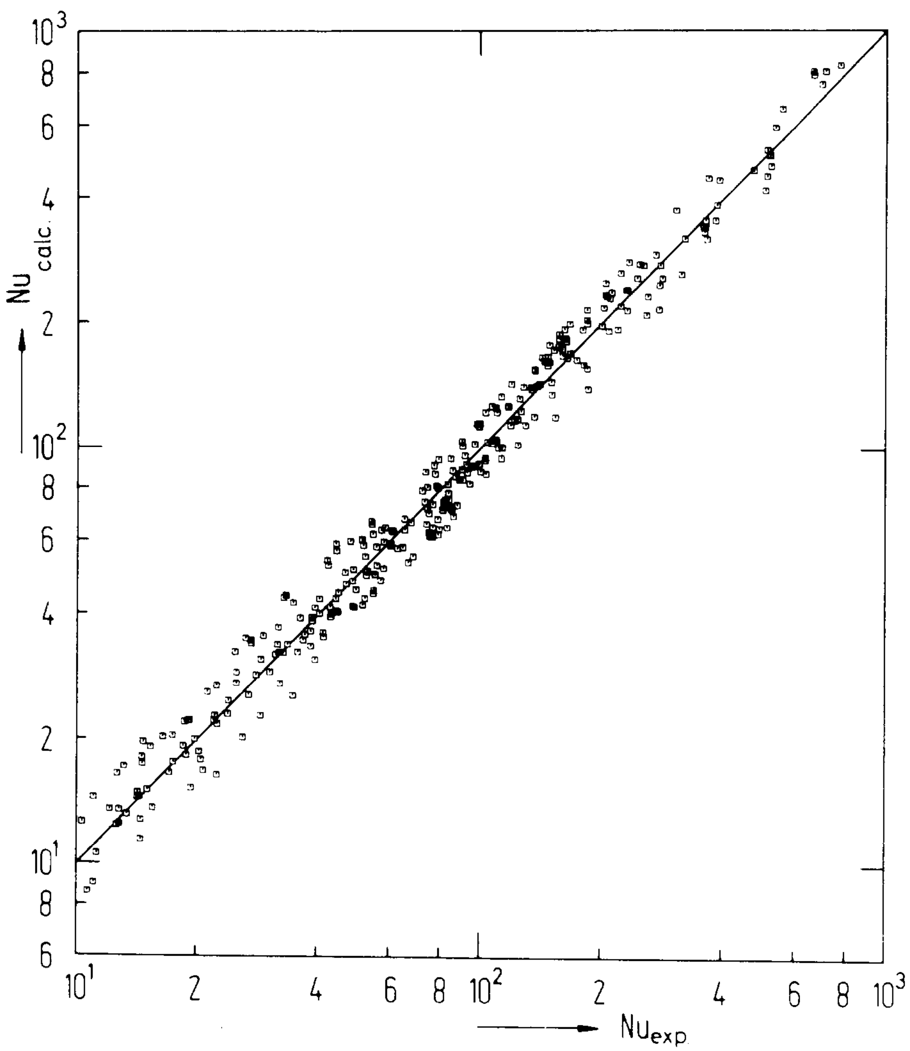

FIG. 4. Nusselt number for water after fourth step of regression analysis.

considered and, in order to permit conclusions on the influence of wall material on heat transfer, the data were further limited to those for which the heating surface material was indicated. These conditions were fulfilled by about 5000 data from 72 papers. Only a few of the above mentioned 5000 data from the literature gave information on the roughness of the heating wall. In cases where these specifications were missing, a mean surface roughness of 1 μm, as is often met in technical applications, was assumed. Very often the experimenters reported their results only with fitted curves accompanied by some of their raw data. In order to have a common basis for the analysis therefore, all the experimental data were fitted by curves $\alpha(\dot{q})$ and each of these curves represented by a certain number (usually four) of characteristic points. Thus the total of 5000 original measuring points were replaced by about 1553 characteristic points. A great number of experimental data are available for water, hydrocarbons, cryogenic liquids and for refrigerants. For each of these substances, or respectively, groups of substances there exists approximately the same number of about 400 characteristic points. It seemed reasonable therefore to study these groups separately, all the more so, as the accuracy of measurement is different for the groups. Heat-transfer data on pool boiling of water are, for instance, more reliable than those on pool boiling of cryogenic liquids. Also, the transport properties of one group of substances may differ considerably from those of another group, whereas the deviations between substances within one group are usually smaller, so some of the dimensionless parameters are different for the various groups of substances. Some of them, important for one group of substances, are expected to be unimportant for another group, an effect which indeed was confirmed by the regression analysis. By considering the groups of substances separately first, equations can be developed with a minimum number of dimensionless variables representing the experimental data within the scope of their accuracy. Eventually an overall-correlation valid for all substances of the four groups was established.

Due to the experimental difficulties, none of the experimenters, when measuring heat-transfer coefficients, simultaneously measured contact angles of the vapour bubbles, so average values of the contact angle β were used for the analysis. It was assumed for water $\beta = 45^\circ$, for refrigerants and hydrocarbons $\beta = 35^\circ$, and for cryogenic liquids $\beta = 1^\circ$. These values were taken from the literature. Contact angles of cryogenic liquids are known to be extremely low. According to Good and Ferry [58] contact angles between liquid hydrogen and stainless steel, inconel, titanium, aluminium, or teflon are zero, whereas Brennan and Skrabek [59] obtained, according to temperature and material of the heating wall, contact angles between 7° and 10° for nitrogen and between 1.5° and 7° for oxygen. They stated, however, that the accuracy of contact angles below 10° is questionable. Bald [60] and Grigorev [40] assumed the contact angles of cryogenic liquids to be zero. As a matter of fact, they are extremely low. However, they cannot vanish, because vapour bubbles form along a heater surface and the surface therefore is not completely wetted. In the analysis the contact angle was assumed arbitrarily to be $\beta = 1^\circ$. When more reliable contact angle data become available the effect of this assumption should be reviewed. A different contact angle leads only to a different constant in the correlation. It does not change the heat-transfer coefficient to be evaluated from the correlation.

In a first run, all the 1553 characteristic points were used in the regression analysis. Upon comparison of the correlation thus obtained with these characteristic points it was apparent that a certain number of characteristic points deviated considerably from the results of the correlation and also from results of other authors.

Eliminating these characteristic points reduced the total to be used to 983 characteristic points representing 2806 original experimental data in a wide pressure range between $0.0001 \leqslant p/p_c \leqslant 0.97$. Details on these experimental data are given in Table 1. From a second analysis with these characteristic points the final correlations were established.

Table 1. Experimental results

(a) Water

Author [Reference]	Geometry	Heater Size (diameter) (cm)	Material roughness	Pressure (bar)
Raben *et al.* [5]	flat plate	$D = 3.77$	pure copper, smoothed and polished	0.0133, 0.0266, 0.0665, 0.2667
Cryder and Finalborgo [6]	cylinder	$D = 3.81$	brass, treated with emery paper	0.0373, 0.141
Nishikawa *et al.* [7]	wire	$D = 0.03$ $L = 4.5$ $= 3.6$	platinum	0.0971, 0.179, 0.318, 1.01, 5.066, 10.132, 20.264, 30.396, 40.528
Styushin and Elinzon [8]	cylinder	$D = 0.08$ $L = 25.0$	brass, polished	0.11, 0.204, 0.296, 0.48
Akin and McAdams [9]	cylinder	$D = 1.6$ $L = 21.59$	nickel coated coppertube	0.157, 0.2919, 0.442, 0.657
König [10]	flat plate		stainless steel electrolyte copper brass MS 58 coated with nickel	0.4, 0.67 1.0133
Rallis and Jawurek [11]	wire	$D = 0.051$	nickel	0.8268
Magrini and Nannei [12]	cylinder	$D = 1.0$ $L = 19.0$	layer of zinc, nickel and tin, polished with emery paper	1.01
Addoms [13]	wire	$D = 0.03$ $= 0.061$ $= 0.122$	platinum	1.01, 53.1, 83.0, 110.4, 136.9
Fedders [14]	cylinder	$D = 1.0$ $t = 0.02$ $L = 15.0$	stainless steel $R_p = 0.475$ $R_p = 3.63\,\mu m$	1.57, 2.65, 4.42, 7.45, 12.74, 19.6, 33.3, 56.8, 98.5

(Table 1.—continued)

(a) Water

Author [Reference]	Geometry	Heater Size (diameter) (cm)	Material roughness	Pressure (bar)
Borishanskii *et al.* [15]	cylinder	$D = 0.694$ $L = 26.0$	stainless steel 1 × 18H9T	4.51, 73.05, 98.1, 147.1, 196.1
Borishanskii *et al.* [16]	cylinder		stainless steel 1 Kh 18N9T clean	5.88, 9.81, 22.6, 31.4, 42.2, 55.0, 99.1, 128.5, 147.0, 169.0, 178.0
Cichelli and Bonilla [17]	flat plate	$D = 9.5$	chromium plated copper, clean and polished	7.93, 18.28, 35.48, 52.76, 70.0
Elrod *et al.* [18]	cylinder	$D = 1.91$ $t = 0.124$ $L = 17.78$	carbon steel monel and inconel commercial material	36.86, 70.0, 106.8

(b) Hydrocarbons

Author [Reference]	Boiling liquid	Geometry	Heater Size (diameter) (cm)	Material, roughness	Pressure (bar)
Mesler and Banchero [19]	benzene ethanol	cylinder	$D = 0.163$ $t = 0.021$	stainless steel	25.294, 7.98, 18.588
Berenson [20]	*n*-pentane	flat plate	$D = 5.08$	copper	1.013
Bonilla and Eisenberg [21]	*n*-heptane	flat plate	$D = 7.62$	chromium plated copper, 0.002 in, clean	0.667, 1.013
Kurihara and Myers [22]	*n*-hexane	cylinder	$D = 7.62$	copper, polished	1.013
Bonilla and Perry [23]	ethanol *n*-butanol	flat plate	$D = 9.093$	chromium plated copper, 0.002 in, clean	0.359, 0.56, 1.772, 1.013
Akin and McAdams [9]	*n*-butanol	cylinder	$D = 1.905$ $L = 21.59$	nickel plated copper, polished	1.013
Fastovskii [24]	benzene		$D = 0.08$ $L = 20.0$	nickel	1.013
Ratiani and Shekriladze [25]	benzene ethanol	cylinder	$D = 0.05$ $L = 6.0$	nickel	1.013 1.013
Miyauchi and Yagi [26]	benzene *n*-hexane	flat plate	$D = 6.8$	brass	1.013 1.013
Borishanskii, Bobrovich and Michenko [15]	ethanol	cylinder	$D = 0.694$ $L = 26.0$	stainless steel 1 × 18H9T	1.013, 3.01, 5.0, 14.71, 39.41, 49.22
Cichelli and Bonilla [17]	benzene *n*-pentane *n*-heptane ethanol	plate		chromium plated copper, 0.002 in, polished clean	3.45, 7.92, 18.24, 31.96, 44.51, 4.137, 7.922, 14.8, 21.96, 28.6, 0.454, 1.013, 3.446, 7.941, 7.931, 1.013, 3.798, 18.24, 35.69, 52.75

Table 1.—continued

(b) Hydrocarbons

Author [Reference	Boiling liquid	Heater Geometry	Size (diameter) (cm)	Material roughness	Pressure (bar)
Huber and Hoehne [27]	benzene	cylinder	$D = 0.953$ $L = 15.24$	stainless steel	2.48, 3.51, 7.94, 13.73, 20.78
Cryder and Gilliland [28]	*n*-butanol	cylinder	$D = 2.64$ $L = 0.833$	brass, clean polished	1.013
Jordan and Leppert [29]	diphenyl *meta*-terphenyl, *ortho*-terphenyl	cylinder	$D = 0.343$ $L = 12.7$	stainless steel, clean	1.013
Cryder and Finalborgo [6]	*n*-butanol	cylinder		brass, roughened with emery paper	1.013

(c) Cryogenic fluids

Author [Reference	Boiling liquid	Heater Geometry	Size (diameter) (cm)	Material roughness	Pressure (bar)
Lyon [30]	helium	ring	$D_0 = 6.861$ $D_i = 6.45$	platin clean and polished	0.063, 1.013
Grigoriev *et al.* [31]	helium	end of a vertical rod	$D = 0.8$ $L = 4.0$	nickel, bronze (Cu + 1,5 Fe), brass (Cu + 30 Zn), copper mean roughness 5–10 μm	1.0
Jergel and Stevenson [32]	helium	cylinder		aluminium (99.9999%)	1.013
Bewilogua *et al.* [33]	hydrogen nitrogen		$D = 0.3$	stainless steel	1.013
Bland *et al.* [34]	nitrogen	rod	$D = 0.2$ $L = 2$	copper with artificial cavities, radius 80–190 μm, depth 150 μm–500 μm	1.013, 2.05, 3.1
Lyon *et al.* [35]	nitrogen oxygen	ring	$D_0 = 6.858$ $D_i = 6.452$	platinum, clean and smoothed	0.432, 2.05, 3.89, 7.822, 15.95, 0.228, 1.018, 2.05, 4.13, 8.04, 15.7
Akhmedov. *et al.* [36]	nitrogen	end of a vertical rod	$D = 1.0$	Cr 18 Ni 9 Ti steel, copper, smoothed with emery paper	1.0, 10.0, 20.0, 25.0, 30.0, 32.0
Marto *et al.* [37]	nitrogen	flat	$D = 2.54$	copper, mirrorfinish	1.013
Frederking [38]	nitrogen	wire	$D = 15–200$ μm	platinum	1.013
Astruc *et al.* [39]	neon	wire coiled in a spiral O.D. 9.5 cm	$D = 0.015$ $L = 49.0$ $A = 2.3\,cm^2$	platinum (pure), smooth	1.0, 10.0, 28.0
Grigorev *et al.* [40]	nitrogen oxygen			copper (Cu + 0.56 Fe) stainless steel (1Cr18Ni9Ti) bronze, brass, roughness 5 μm	1.013 1.013

Table 1.—continued

(c) Cryogenic Fluids

Author [Reference]	Boiling fluid	Geometry	Heater Size (diameter) (cm)	Material roughness	Pressure (bar)
Kosky and Lyon [41]	nitrogen oxygen methane argon	circular end of a cylinder	$D = 1.9$	platinum coated with ETP-copper, clean, polished	3.58, 7.587, 15.7, 23.0, 29.78, 32.82, 42.85, 1.08, 16.4, 1.08, 33.33
Ackermann *et al.* [42]	nitrogen			German silver smooth, depth = 0.2 μm	1.013
Lyon [43]	nitrogen oxygen	cylinder	$D = 0.952$–6.98 $L = 4.4$–10.4	copper, coated with gold, clean polished, (1–4 μm)	1.013
Haselden and Peters [44]	oxygen	cylinder	$D = 1.588$ $L = 7.62$	copper, clean	1.013
Sciance *et al.* [45]	methane	cylinder	$D = 2.06$ $L = 10.16$	ARMCO-iron, coated with gold	41, 76
Sciance *et al.* [46]	ethane	cylinder	$D = 2.06$ $L = 10.16$	ARMCO-iron, coated with gold	4.89, 14.67
Bewilogua *et al.* [47]	nitrogen helium hydrogen	flat	$A = 2.9\ cm^2$ $A = 4.9\ cm^2$ $A = 2.9\ cm^2$	copper, smoothed with emery paper, depth ≈ 0.2 μm	0.983, 2.94 1.0 1.013, 9.8

(d) Refrigerants

Author [Reference]	Boiling fluid	Geometry	Heater Size (diameter) (cm)	Material roughness	Pressure (bar)
Hesse [48]	R 12 R 114 R 113	tube	$D = 1.4$ $L = 35.0$	pure nickel (99.8%) $R_p = 0.61$	7.0, 14.0, 30.0, 3.0, 6.0, 9.0, 12.0, 15.0, 20.0, 0.5, 1.0
Wickenhäuser [49]	R 113 RC 318	tube	$D = 0.08$ $L = 27.0$	copper, $R_p = 0.4$ $R_p = 0.9$	1.02, 1.55, 3.21, 3.65, 7.03, 13.82
Stephan [1]	R 11	tube and flat plate	$D = 3$ $L = 50$	copper, $R_p = 7.9$, 4.4, 1.4, 0.51, 0.15 μm	1.31
Gorenflo [50]	R 11 R 113	tube	$D = 1.2$	copper commercial, $R_p = 0.4$ μm smooth	1.3, 2.0, 3.0, 0.1, 0.4
Hesse [51]	R 114	tube	$D = 3$ $L = 47.0$	copper, $R_p = 0.2$ μm polished	0.37, 1.28, 2.52
Schroth [52]	R 11 R 12	tube	$D = 2.5$ $L = 40.0$	steel, $R_p = 9.0$ μm	0.537, 1.22, 2.24, 2.0, 2.67, 5.02
Stephan [53]	R 12	flat plate	$D = 13.0$ $t = 2.5$	copper $R_p = 1.0$ μm	1.63, 2.35, 3.51, 5.02
Happel [54]	R 113	tube	$D = 1.4$ $t = 0.075$ $L = 40.0$	99.8 nickel $R_p = 0.43$ μm	1.0

Table 1. – continued

(d) Refrigerants

Author [Reference]	Boiling fluid	Geometry	Heater Size (diameter) (cm)	Material roughness	Pressure (bar)
Danilova and Kupriyonova [55]	R 21 RC 318	tube	$D = 0.51$ $L = 9.1$	stainless steel $R_p = 3.84$, 9.1 μm	0.71, 1.04, 2.15, 3.65, 13.82
Danilova [56]	R 12 R 22 R 113	tube	$D = 1.25$ $t = 0.02$ $L = 25.3$	stainless steel, commercial material	5.68, 9.59, 3.55, 4.99, 6.81, 9.104, 15.32, 24.24, 1.09
Sciance *et al.* [45]	propane *n*-butane	cylinder	$D = 2.06$ $L = 10.16$	ARMCO-iron coated with gold	2.13, 4.26, 8.52, 12.78, 21.3, 25.56, 1.9, 5.7, 11.4
Abadzic [57]	carbon-dioxide	wire	$D = 0.01$ $L = 10.0$	pure platinum	55.7–57.4

4. RESULTS

On this basis, the following equations were obtained, the dimensionless parameters being arranged in the order of their influence on the Nusselt number.

For *water:*

$$Nu = 0.246 \cdot 10^7 X_1^{0.673} X_4^{-1.58} X_3^{1.26} X_{13}^{5.22} \tag{6}$$

$10^{-4} \leqslant p/p_c \leqslant 0.886$, contact angle $\beta = 45°$,
mean absolute error 11.3%.

For *hydrocarbons:*

$$Nu = 0.0546 (X_5^{0.5} \cdot X_1)^{0.67} X_{13}^{-4.33} X_4^{0.248} \tag{7}$$

$5.7 \cdot 10^{-3} \leqslant p/p_c \leqslant 0.9$, contact angle $\beta = 35°$,
mean absolute error 12.2%.

For *cryogenic fluids:*

$$Nu = 4.82 X_1^{0.624} X_9^{0.117} X_5^{0.257} X_3^{0.374} X_4^{-0.329} \tag{8}$$

$4 \cdot 10^{-3} \leqslant p/p_c \leqslant 0.97$, contact angle $\beta = 1$,
mean absolute error 14.3%.

For *refrigerants:*

$$Nu = 207 X_1^{0.745} X_5^{0.581} X_6^{0.533} \tag{9}$$

$3 \cdot 10^{-3} \leqslant p/p_c \leqslant 0.78$, contact angle $\beta = 35°$,
mean absolute error 10.57%.

For *all substances* used in the analysis:

$$Nu = 0.23 X_1^{0.674} X_5^{0.297} X_4^{0.371} X_{13}^{-1.73} X_2^{0.35} \tag{10}$$

$10^{-4} \leqslant p/p_c \leqslant 0.97$,
mean absolute error 22.3%.

The mean absolute error gives the mean absolute deviation from the characteristic points used for the final analysis.

For many applications the accuracy of equation (9) is not adequate. One should prefer therefore the individual equations (5)–(8).

Only some of the 13 original dimensionless numbers appear in the above equations, namely:

$$X_1 = (\dot{q}d)/(\lambda' T_s); \quad X_2 = (a'^2 \rho')/(\sigma d);$$
$$X_3 = (c'_p T_s d^2)/a'^2; \quad X_4 = (r d^2)/a'^2;$$
$$X_5 = \rho''/\rho'; \quad X_6 = \nu'/a';$$
$$X_9 = (\rho c_p \lambda)_c/(\rho' c'_p \lambda'); \quad X_{13} = (\rho' - \rho'')/\rho'.$$

In order to facilitate the practical application of equations (6) to (8), we employ the simpler form $\alpha = c\dot{q}^n$, where c depends on the thermal properties of the substances and may be represented as a function of pressure. The value of n is different for each group of substances. We have therefore with α in W/m$^2 \cdot$K, $\dot{q}$ in W/m^2:

For *water:*

$$\alpha = c_1 \dot{q}^{0.673} \tag{11}$$

For *hydrocarbons:*

$$\alpha = c_2 \dot{q}^{0.670}. \tag{12}$$

For *cryogenic fluids*, where the heat-transfer coefficients proved to depend also on the material of the cover or heater surface

$$\alpha = c_3 \dot{q}^{0.624} (\rho c_p \lambda)_c^{0.117} \tag{13}$$

where ρ in kg/m^3, c_p in kJ/kg K) and λ in W/(Km) are evaluated at the saturation temperature of the boiling liquid. Replacing equation (12) by

$$\alpha = c'_3 \dot{q}^{0.624} \tag{13a}$$

with $c'_3 = c_3 (\rho c_p \lambda)_c^{0.117}$ we find for atmospheric pressure $p = 1$ bar, and for different combinations of heater surface and boiling liquid, the values c'_3 listed in Table 2.

For *refrigerants* we have:

$$\alpha = c_4 \dot{q}^{0.745} \tag{14}$$

The pressure dependent values c_1, c_2, c_3, c_4 are plotted in Figs. 5–8 for different substances, thus permitting a simple and rapid evaluation of heat-

Table 2. Values c_3' for different combinations of heater surface and boiling liquid; $p = 1$ bar

Combination Heater surface/boiling liquid	c_3'
Copper/nitrogen	12.65
Stainless steel/nitrogen	7.6
Copper/oxygen	12.3
Stainless steel/oxygen	7.46
Copper/hydrogen	38.9
Stainless steel/hydrogen	21.0

transfer coefficients of many substances important for technical applications.

As already mentioned the mean surface roughness R_p in the equations (5)–(13) was assumed to be $R_p = 1$ μm. As shown in an earlier paper [1], in a first approximation the heat-transfer coefficient α is proportional to $R_p^{0.133}$ for surfaces with a regular roughness distribution as prepared for example with emery paper, on a lathe or on a drawing bench. The surface roughness therefore may be taken into account by multiplying the heat-transfer coefficients

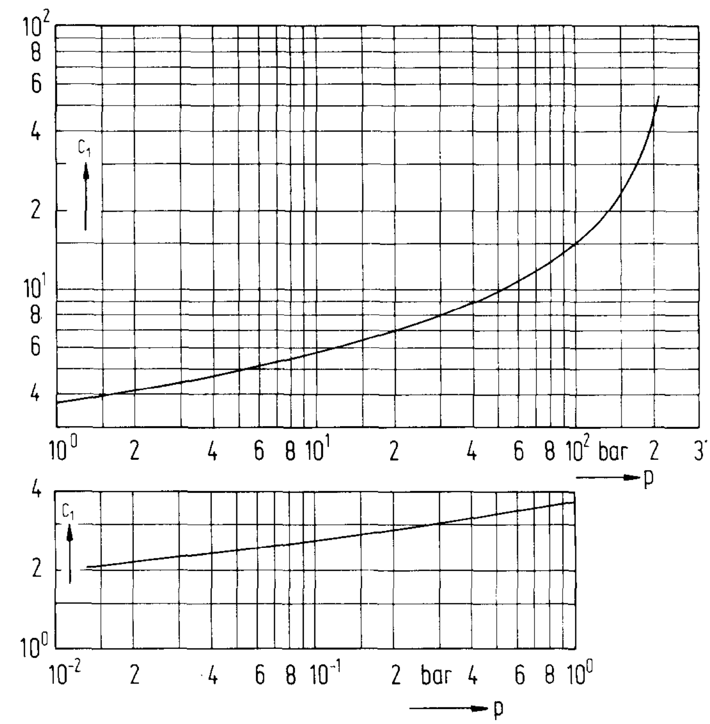

FIG. 5. Constant c_1 in equation (10).

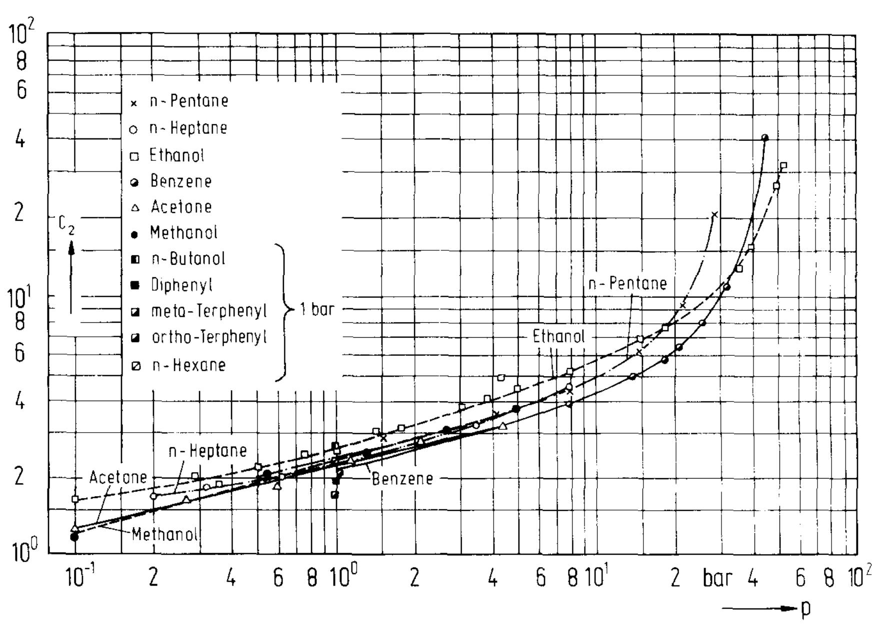

FIG. 6. Constant c_2 in equation (11).

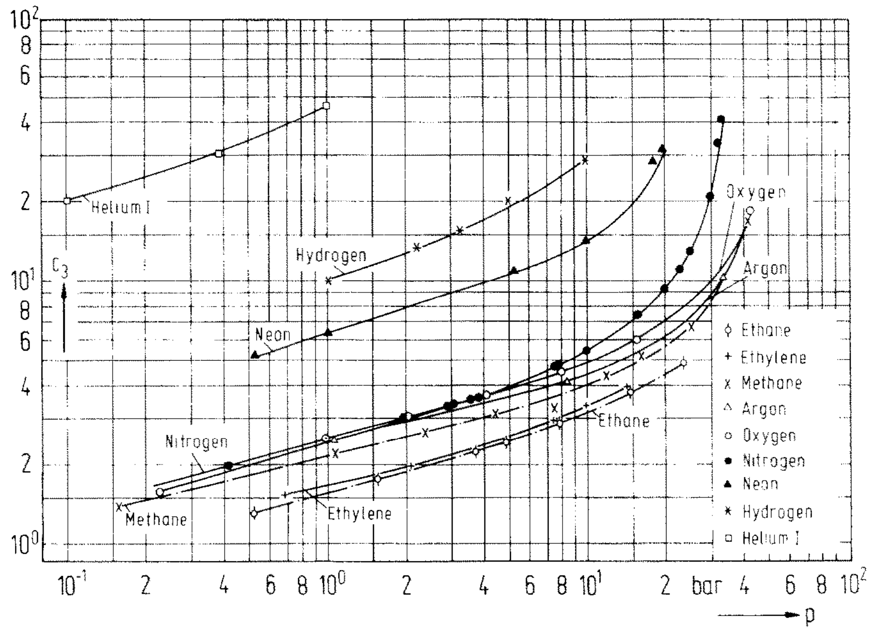

FIG. 7. Constant c_3 in equation (12).

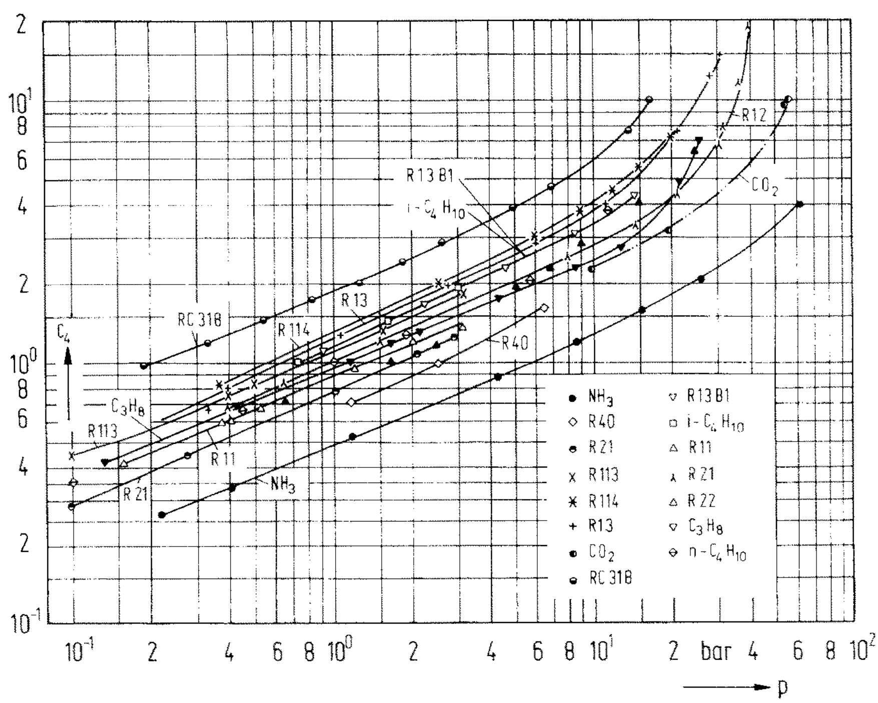

FIG. 8. Constant c_4 in equation (13).

from equations (5)–(13), for $0.1 \leqslant R_p \leqslant 10\,\mu m$, with a factor $R_p^{0.133}$, R_p in μm.

It is noteworthy that in the above equations only the equations for boiling of cryogenic liquids include a term for the thermal properties of the heater surface or the cover protecting the wall of the heater. Grigorev *et al.* [40] stated already that the wall material has a pronounced influence on heat transfer in boiling of cryogenic liquids. They point out that heat-transfer coefficients with boiling nitrogen on different metal surfaces differ by more than a factor of 10, and with boiling of helium by more than a factor of 40, whereas a much lower influence of the heater surface has been noted in boiling of normal liquids. Grigorev demonstrated that this effect may be explained by different factors: The thermophysical properties of various metals, such as heat conductivity and heat capacity, differ significantly more than at normal temperature. A small change in the boiling heat flux at cryogenic temperature and hence the wall temperature of the heater, therefore causes a considerable change of the thermal properties of the heater.

Furthermore the thermal properties of different heater materials differ much more at low temperatures. Another effect may come from the extremely small contact angles between boiling liquid and heater wall, which though very small, may also differ considerably for different materials. Finally one should also keep in mind that most of the cryogenic liquids exhibit a higher heat conductivity than liquids with a higher boiling point. The thermal resistance of the heater therefore is more important in boiling of cryogenic liquids.

As a concluding remark one should notice that the cited equations allow a fairly good representation of the existing experimental data. They should, however, not be regarded as conclusive but be improved as soon as a sufficient number of new and more accurate data become available.

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EXPRESSIONS DU TRANSFERT THERMIQUE EN EBULLITION AVEC CONVECTION NATURELLE

Résumé—Il n'existe pas actuellement de théorie explicative permettant la prévision des coefficients de transfert thermique pour l'ébullition avec convection naturelle, malgré de nombreux efforts dans ce domaine. Afin d'établir des formules ayant une large application, on applique les méthodes d'analyse de régression à 5000 points expérimentaux pour l'ébullition avec convection naturelle.

Ces données peuvent être regroupées en quatre familles (eau, hydrocarbures, fluides cryogéniques et réfrigérants) et en employant un système différent de nombres sans dimension pour chaque groupe de substances. On peut établir une équation valable pour toutes les substances mais sa précision est moindre que celle des formules individuelles sans ajouter une quelconque complexité.

GLEICHUNGEN FÜR DEN WÄRMEÜBERGANG BEIM VERDAMPFEN IN NATÜRLICHER STRÖMUNG

Zusammenfassung—Trotz vieler Bermühungen ist es bisher nicht gelungen, eine umfassende Theorie zur Vorausberechnung des Wärmeübergangs beim Verdampfen in natürlicher Strömung zu entwickeln. Um Korrelationen mit möglichst breitem Gültigkeitsbereich zu erhalten, wurden die Methoden der Regressionsanalyse auf die etwa 5000 bisher bekannten Meßdaten über den Wärmeübergang beim Verdampfen in natürlicher Strömung angewandt. Wie sich dabei zeigte, lassen sich diese Daten am besten wiedergeben, wenn man die Stoffe in vier Gruppen (Wasser, Kohlenwasserstoffe, tiefsiedende Fluide und Kältermittel) einteilt und einen unterschiedlichen Satz dimensionsloser Größen für jede dieser Stoffgruppen verwendet, da einige der dimensionslosen Größen für eine Stoffgruppe wichtig, für eine andere hingegen unbedeutend sein konnen. Es konnte außerdem eine einzige Gleichung für alle Stoffe angegeben werden, deren Genauigkeit jedoch geringer ist als die der Gleichungen für die einzelnen Stoffklassen, solange man auch für diese allgemeine Gleichung keinen unerwünscht komplizierten Ansatz wählt.

ОБОБЩЕННЫЕ СООТНОШЕНИЯ ДЛЯ ТЕПЛООБМЕНА ПРИ КИПЕНИИ В УСЛОВИЯХ ЕСТЕСТВЕННОЙ КОНВЕКЦИИ

Аннотация — До настоящего времени, несмотря на неоднократные попытки, не разработана теория, которая позволила бы рассчитывать коэффициенты теплообмена при кипении в условиях естественной конвекции. С целью получения обобщенных соотношений с широкой областью применения использовались методы регрессивного анализа для почти 5000 экспериментальных точек, относящихся к теплообмену при кипении в условиях естественной конвекции. Анализ показывает, что лучше всего эти данные обобщаются в том случае, если провести подразделение исследуемых веществ на четыре группы (вода, углеводороды, криогенные жидкости и хладагенты) и для каждой группы веществ использовать различные сочетания безразмерных критериев. Это связано с тем, что одни из критериев, являющиеся важными для одной группы, несущественны для другой. Можно построить одно уравнение, которое было бы справедливым для всех веществ, но оно было бы менее точным и более сложным, чем отдельные обобщенные соотношения.